Theodolite Observations and Least Squares

ABSTRACT

This paper examines the reduction of theodolite direction observations in connection with subsequent least squares adjustments. These reductions of direction measurements are often called station adjustments. Some of the reduction procedures are reviewed. Whether to use grand means, single arcs or half arcs in a least squares adjustment of a survey network is considered with regard to systematic instrument errors, electronic data recording equipment, a priori standard deviations and computer resources. The effect on the statistical testing of survey networks is also discussed.

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1. Reduction of Direction Measurements (Station Adjustment)

1.1 Reduction of Full Arcs (Traditional Procedure)

A widely adopted procedure when analysing direction measurements is to reduce FL (face or circle left, measured clockwise) and FR (face right, measured anticlockwise) observations to a mean and then find the grand mean from all arcs (sets) at that site. This method has been used for many years. One early reference is Jordan (1893). This 'station adjustment' is a least squares adjustment of the individual arcs that solves for directions and orientations. An example of these calculations is shown in Table 1. The example is given to show the mechanics of the calculations and for comparison with other methods. As usual, the zeros of circle and micrometer are changed between arcs.

The standard deviation of a grand mean direction is then calculated from:

$$s_T = \sqrt{\frac{\Sigma v^2}{a\,(a-1)\,(t-1)}} \qquad (1)$$

where a is the number of arcs and t is the number of targets in an arc. The residuals v are calculated in two steps. First, v' is the difference of a reduced mean (of FL and FR) observation from the grand mean direction. Then v is calculated from $v = v' - \Sigma v'/t$, where $\Sigma v'$ is the sum of the v' for the arc. $\Sigma v'/t$ is the orientation unknown of an individual arc.

The standard deviation of a single mean direction (mean of FL and FR) is calculated from:

$$s_S = \sqrt{\frac{\Sigma v^2}{(a-1)\,(t-1)}} \qquad (2)$$

Arc	Target	Face Left			Diff	Face Right			Mean			Reduced Mean			v'	v	v2
	Omega	0	00	06	+3	180	00	09	0	00	07.5	0	00	00	0.0	2.0	3.9
	T.4	21	46	29	+4	201	46	33	21	46	31	21	46	23.5	-1.8	0.2	0.1
1	Astro	63	17	21	+5	243	17	26	63	17	23.5	63	17	16	-1.9	0.1	0.0
	Wild	100	24	01	+4	280	24	05	100	24	03	100	23	55.5	-3.4	-1.4	2.0
	RA	142	10	53	-5	322	10	48	322	10	50.5	142	10	43	-2.9	-0.9	0.8
														Σ	-9.9	0.0	
	Omega	45	02	08	+5	225	02	13	45	02	10.5	0	00	00	0.0	-1.8	3.3
	T.4	66	48	26	+9	246	48	35	66	48	30.5	21	46	20	1.8	-0.1	0.0
2	Astro	108	19	17	+6	288	19	23	108	19	20	63	17	09.5	4.6	2.8	7.8
	Wild	145	26	00	0	325	26	00	145	26	00	100	23	49.5	2.6	0.8	0.6
	RA	187	12	55	-9	7	12	46	187	12	50.5	142	10	40	0.1	-1.7	2.9
														Σ	+9.1	0.0	
	Omega	90	05	07	+3	270	05	10	90	05	08.5	0	00	00	0.0	-0.2	0.1
	T.4	111	51	33	-2	291	51	31	111	51	32	21	46	23.5	-1.8	-2.0	3.9
3	Astro	153	22	24	-2	333	22	22	153	22	23	63	17	14.5	-0.4	-0.6	0.4
	Wild	190	28	55	+4	10	28	59	190	28	57	100	23	48.5	3.6	3.4	11.6
	RA	232	15	48	+2	52	15	50	232	15	49	142	10	40.5	-0.4	-0.6	0.4
														Σ	+1.1	0.0	
	Omega	135	07	10	12	315	07	22	135	07	16	0	00	00	0.0	0.1	0.0
	T.4	156	53	38	-4	336	53	34	156	53	36	21	46	20	1.8	1.8	3.3
4	Astro	198	24	29	+7	18	24	36	198	24	32.5	63	17	16.5	-2.4	-2.3	5.3
	Wild	235	31	11	0	55	31	11	235	31	11	100	23	55	-2.9	-2.8	7.8
	RA	277	17	51	+4	97	17	55	277	17	53	142	10	37	3.1	3.2	10.2
														Σ	-0.4	0.0	

$\Sigma v^2 = 64.4$

Target	**Grand Means**		
Omega	0	00	00.0
T.4	21	46	21.8
Astro	63	17	14.1
Wild	100	23	52.1
RA	142	10	40.1

Number of arcs $a = 4$;

Number of targets $t = 5$

Standard Deviation of Grand Mean Direction: $s_T = \sqrt{64.4/4\,(4-1)(5-1)} = \pm 1.16''$

Table 1. Example of traditional station adjustment procedure and error analysis for horizontal direction measurements. [Originally prepared by K.I.Groenhout, January 1980 and subsequently modified by the authors.]

The grand mean directions can then be entered into a least squares adjustment of a network. This has been done since C.F. Gauss used the procedure with equally weighted directions in the Triangulation of Hanover 1820–1830 (Jordan, 1888). In the least squares adjustment of the survey net one orientation parameter is determined for this set of grand mean directions. The a priori standard deviation assigned to the grand mean directions is either derived from s_T above or an estimate of the population standard deviation of the observations is used. (Different populations apply for different instruments, observers and observing conditions). In the least squares process the standard deviation of a direction is usually increased to include effects of centring errors.

1.2 Reduction of Half Arcs (Richardus' Procedure)

Another method has been described by Richardus (1984). In this case FL and FR observations are not combined. Each face is treated separately. Richardus advises to shift the theodolite circle between faces and between arcs. The residuals are calculated by taking the difference between a single face observation (reduced to the RO) and the grand mean. An example of these calculations is shown in Table 2. Note that Table 2 uses the same observations as Table 1.

Arc	Target	Face	Obs			Red.	to	RO	vR'	vR	vR2	Grand		Means
1	Omega	L	0	00	06	0	00	00	0.0	2.4	5.6			
	T.4		21	46	29	21	46	23	−1.3	1.1	1.3			
	Astro		63	17	21	63	17	15	−0.9	1.5	2.3			
	Wild		100	24	01	100	23	55	−2.9	−0.5	0.3			
	RA		142	10	53	142	10	47	−6.9	−4.5	20.3			
								Σ	−11.9	0.0				
1	Omega	R	180	00	09	0	00	00	0.0	1.6	2.5			
	T.4		201	46	33	21	46	24	−2.3	−0.7	0.5			
	Astro		243	17	26	63	17	17	−2.9	−1.3	1.7			
	Wild		280	24	05	100	23	56	−3.9	−2.3	5.3			
	RA		322	10	48	142	10	39	1.1	2.7	7.3			
								Σ	−7.9	0.0				
2	Omega	L	45	02	08	0	00	00	0.0	−0.4	0.2			
	T.4		66	48	26	21	46	18	3.8	3.3	11.1			
	Astro		108	19	17	63	17	09	5.3	4.7	22.1			
	Wild		145	26	00	100	23	52	0.1	−0.3	0.1			
	RA		187	12	55	142	10	47	−6.9	−7.3	53.3			
								Σ	2.1	0.0				
2	Omega	R	225	02	13	0	00	00	0.0	−3.2	10.4			
	T.4		246	48	35	21	46	22	−0.3	−3.5	12.1			
	Astro		288	19	23	63	17	10	4.1	0.9	0.8			
	Wild		325	26	00	100	23	47	5.1	1.9	3.6			
	RA		7	12	46	142	10	33	7.1	3.9	15.2			
								Σ	16.1	0.1				
3	Omega	L	90	05	07	0	00	00	0.0	0.8	0.6			
	T.4		111	51	33	21	46	26	−4.3	−3.5	12.1			
	Astro		153	22	24	63	17	17	−2.9	−2.1	4.4			
	Wild		190	28	55	100	23	48	4.1	4.9	24.0			
	RA		232	15	48	142	10	41	−0.9	−0.1	0.0			
								Σ	−3.9	0.0				
3	Omega	R	270	05	10	0	00	00	0.0	−1.2	1.5			
	T.4		291	51	31	21	46	21	0.8	−0.5	0.2			
	Astro		333	22	22	63	17	12	2.1	0.9	0.8			
	Wild		10	28	59	100	23	49	3.1	1.9	3.6			
	RA		52	15	50	142	10	40	0.1	−1.1	1.2			
								Σ	6.1	0.0				
4	Omega	L	135	07	10	0	00	00	0.0	4.2	17.4			
	T.4		156	53	38	21	46	28	−6.3	−2.1	4.3			
	Astro		198	24	29	63	17	19	−4.9	−0.7	0.5			
	Wild		235	31	11	100	24	01	−8.9	−4.7	22.1			
	RA		277	17	51	142	10	41	−0.9	3.3	10.9			
								Σ	−20.9	0.0				
4	Omega	R	315	07	22	0	00	00	0.0	−4.0	16.2	0	00	00.0
	T.4		336	53	34	21	46	12	9.8	5.7	32.8	21	46	21.8
	Astro		18	24	36	63	17	14	0.1	−3.9	15.2	63	17	14.1
	Wild		55	31	11	100	23	49	3.1	−0.9	0.8	100	23	52.1
	RA		97	17	55	142	10	33	7.1	3.1	9.6	142	10	40.1
								Σ	20.1	0.0	358.2 = Σv^2			

$h = 8$; $t = 5$ $s_R = \sqrt{358.2/8\ (8-1)\ (5-1)} = \pm 1.26''$

Table 2 Example of reduction of horizontal direction measurements by Richardus' method.

The standard deviation of a grand mean direction is then calculated from

$$s_R = \sqrt{\frac{\Sigma v_R^2}{h\,(h-1)\,(t-1)}} \qquad (3)$$

where h is the number of half arcs or faces observed and t is the number of targets in an arc. The residuals v_R are calculated in two steps similarly to the v in the traditional method. Even though h = 2a there is no simple, general relationship between s_T and s_R because the residuals v_R contain noise/errors that are not represented in the residuals v.

This technique inherently assumes that any systematic difference between FL and FR is constant and therefore absorbed into orientation parameters. If there is a variable systematic difference between FL and FR, then this calculation is not valid. Table 3 and Section 2 below discuss theodolite errors further.

Again the grand mean directions are then entered into a least squares adjustment. In the least squares adjustment of the survey net one orientation parameter is determined for this set of grand mean directions. The a priori standard deviation assigned to the grand mean directions is either derived from s_R above or an estimate of the population standard deviation of the observations is used. Again terms are often added to account for centring errors.

1.3 Comparison of Precisions

In either of the above procedures s_T or s_R can be statistically tested against a population standard deviation, if it is known. (Again note that population parameters vary with observers, instrument and observing conditions). It is possible to use a global a priori standard deviation for all direction observations in a net. Alternatively, individual standard deviations can be assigned on a site-by-site basis. Note that in the sample data (Tables 1 and 2) s_T does not equal s_R. We suspect that s_T will usually be less than s_R because any systematic and random errors (signal and noise) are reduced by taking the mean of two faces.

2. Review of Theodolite Errors

Table 3 lists most known systematic error sources in direction measurements and their mathematical descriptions. It would exceed the scope of this paper to elaborate on the likely magnitudes of the instrumental errors. They vary considerably between different makes and types of theodolites. In the case of electronic theodolites and tacheometers, manufacturers compensate errors increasingly by factory calibration and real-time application of corrections by the on-board microprocessor.

The table distinguishes between single index and double index theodolites. The former read the horizontal circle at one point only while the latter read the circle at diametrically opposite points. Because the actual field procedures also influence the error scenario, some aspects are included in Table 3 as far as relevant to direction measurements. It can be seen that the difference between FL and FR readings is not a constant. The twelve direction errors e depend on the magnitude (and azimuth) of a number of primary instrument errors and some other variables such as zenith angle (6 times), distance to target (2 times) and time. In consequence, the difference between FL and FR direction measurements also depends on zenith angle, distance, azimuth and time. Most of the errors (7 out of 12) are cancelled when measuring in two faces and taking the mean. Errors 7 and 8 can be reduced by measuring multiple arcs on different circle and micrometer zeros. Errors 9 and 10 can be reduced through the use of electronic theodolites with dual-axes compensators.

Considering these two aspects, namely that the difference between FL and FR is not a constant and the mean of FL and FR eliminates several errors, leads us to recommend the use of mean of FL and FR readings rather than single face observations. To remove any biasing of the standard deviations derived from half-arc reductions, the systematic errors could be solved for by a more general least squares station adjustment or a more general network adjustment. In doing so, the number of unknown parameters could easily increase by 12–24 and, thus, severely decrease the degree of freedom. (Some of the additional parameters vary from station to station (e.g. b, ϵ, i_o, α in Table 3), others may be assumed constant for the period of a network measurement with one theodolite.) It is unlikely, that all measured arcs would permit a solution for all systematic errors due to insufficient spread and unfavourable distribution over zenith angles, distances and azimuth.

3. Directions in Least Squares Estimations

3.1 A Priori Standard Deviations

As well as entering the observations or their derived means in a least squares adjustment of a network, the standard deviation of the data also have to be entered. Depending on the user and the particular survey involved, different methods to obtain the standard deviation can be chosen. This paper does not argue the relative merits of using a supposedly known population value compared to using a value derived from the sample involved. The standard deviation could be based on experience, from a calculation such as s_T and s_R above, or from some other calculation from the data set, such as miscloses around loops or variance component estimation.

If a population standard deviation obtained from journal articles or manufacturers is used then be sure it is calculated by the appropriate method. (Manufacturers' specifications are now usually based on DIN 18723 which uses s_s of the traditional method as in Table 1). That is, if grand means are entered then a different value of standard deviation should be used to that used for means of single arcs. A similar argument applies to using single face observations. Again we say that different methods give different standard deviations for the same observations.

3.2 Patterson's Recommendations

In two recent papers (Patterson, 1987 & 1988) the above methods have been questioned regarding the appropriate number of orientation parameters. Patterson argues that a separate orientation parameter is required for each arc rather than one orientation parameter for the grand mean set. He also states that it may be necessary to treat each face of observations as the input data, each with their own orientation parameter. However he uses Richardus' method for determining the standard deviations of observations. He does not discuss the effect of instrument and measurement errors on single face observations in detail.

3.3 Use of Grand Means

Calculating a grand mean of directions for least squares input purposes is an example of adjustment in phases. It is well known that adjustments in phases produce the same results as complete adjustments provided the calculations are done correctly with all variances and covariances carried through the adjustment. So coordinates of network points will not change if separate arcs or even separate face observations are entered compared to a solution using grand means of the directions provided the same orientations, coordinates and other parameters are used in each adjustment. However, the error ellipses and related output might change. The degrees of freedom of the network adjustment changes considerably.

A station adjustment of directions requires an arbitrary orientation to be held fixed. This can be done by use of a constraint equation so that the overall change in orientation for all arcs is zero (see for example, Van Gysen, 1990) or by reparameterisation (Patterson, 1987). The traditional solution of reducing the first ray to $0° 00' 00''$ is used in Figures 1 and 2. The appropriate input standard deviation for input or testing against population values is s_T. When using the grand mean directions, the errors 1, 2, 4–6, 11, 12 of Table 3 have no effect. Furthermore, the effects of errors 7 and 8 are greatly reduced by changing zeros of circle and micrometer between arcs. Electronic theodolites with dual—axes compensators permit on-line compensation of errors 9 and 10. It should be noted, however, that the standard deviation s_T is affected fully by errors 3, 7, 8 and, usually, 9 and 10.

Description of Error	Effect on Single Direction Measurement	Affects: Single Index	Double Theodolite	Elimination By FL & FR Mean?	Remedies, Remarks Definitions
1 average trunnion axis not perpendicular to vertical axis by angle i_T	$e = i_T \cot z$	yes	yes	yes	i_T = trunnion axis inclination
2 mean line of collimation not perpendicular to trunnion axis by angle i_C	$e = i_C / \sin z$	yes	yes	yes	(z = zenith angle) i_C = collimation error
3 wobble of trunnion axis about its mean position	i_T = Fourier series of z i_C = Fourier series of z	yes	yes	somewhat (hor sights)	(λ_o = 360° of z)
4 mean line of collimation not intersecting vertical axis	$e'' = (E/D)\ 206265''$	yes	yes	yes	D = dist to target E = eccentricity
5 wobble, wander of focussing lens	E = f (D) i_C = f (D)	yes	yes	yes	depends on position of focussing lens
6 centre of circle graduation eccentric (by d) to axis of rotation	$e = (d / R) \sin \alpha$	yes	no	yes	R = Radius of circle graduations α = azimuth from direction of eccentricity
7 circle graduation errors: • long periodic • short periodic	 e = Fourier series of β $e = f(\beta)$	 yes (λ_o = 360°) yes	 less (λ_o = 180°) yes	 somewhat no	B = circle reading λ_o = zero order wavelength of Fourier series
8 micrometer or interpolation errors	$e = f(\gamma)$	yes	yes	no	γ = micrometer electronic reading
9 dislevelment i_o of average position of vertical axis (δ = azimuth from direction of maximum dislevelment)	$e = i_o \sin \delta \cot z$	yes	yes	no	real-time correction possible with dual-axes'compensators in electronic theodolites
10 wobble of vertical axis about its average position	i_o = Fourier series of ϵ (Kern: λ_o = 120° of ϵ) (others: λ_o = 720° of ϵ)	yes	yes	no	ϵ = orientation of rotating relative to non-rotating part of theodolite (also see comment in 9)
11 tripod twist at time t_i (to = time at start of arc)	$e = b\,(t_i - t_o)$	yes	yes	yes	b = linear trend with time of twist
12 additive constant a of dual-axes compensator	$e = a \sin \delta \cot z$	yes	yes	yes	a = unknown residual part of add. constant

Table 3: Summary of errors e in measured horizontal directions due to errors of the theodolite, its state of levelling and its set-up (expanded from Cooper 1982 and Kahmen & Faig 1988)

3.4 Use of Means of Single Arcs

With the current ease of computing a least squares adjustment it is possible to include each arc of observations in a network adjustment. A separate orientation parameter is then determined for each arc. No constraint equation is required. In principle no prior station adjustment is required nor is it necessary to reduce directions so that 0° is the value for the first direction. (Station adjustments may however be carried out for quality assurance purposes and to get s_s.) The mean of FL and FR can be used regardless of the initial circle setting. The appropriate standard deviation for least squares input or testing against population values is s_S.

Using the single arc means will yield more information (than grand means) in the residuals for outlier analysis and an analysis of errors which change from one arc to another. The means of single arcs as well as their standard deviations are fully affected by the errors 3, 7, 8 (and often 9, 10) of Table 3.

3.5 Use of Half Arcs

Using raw (single face) observations reveals the real noise level and residuals are fully exposed to systematic errors whereas taking means dampens/filters this noise. If single face observations are entered into least squares adjustments instead of the mean of FL and FR observations then either corrections are required for some of the errors described in Table 3 or extra parameters should be determined in the least squares adjustment. Failure to do so leads to erroneous a posteriori accuracies of the directions and erroneous accuracies of all adjusted parameters (including coordinates).

If extra parameters are included to model systematic errors then the coordinates might change. Extra parameters in an adjustment usually yield smaller residuals for the observations but care needs to be taken to avoid over-parameterisation. Extra parameters for the theodolite errors will probably only be feasible in research work where they are of particular interest or the greatest possible accuracy is required. In such cases the observation program can be extended to gather enough and well distributed observations for the determination of extra parameters. In normal practice they will cause an increase in computer resources needed. Neither s_R or $s_R \sqrt{h}$ are suitable as input standard deviations in the presence of variable systematic errors, as discussed in Section 1.2 above.

3.6 Computing Considerations

Modern electronic data recorders for theodolites make it easy to enter raw single face observations into a least squares adjustment of a network. It is no longer essential for practical ease to preprocess directions to produce grand means for input. However, extra observations and parameters require more computer space and time to solve a net. Also it is necessary to ensure a matching pair of FL and FR observations. Data recorders can often be programmed so that means are recorded instead of, or as well as, the individual pointings.

Increasing the number of observations and the number of parameters considerably increases the computing resources required. It is well known that the increase is greater than linear. Details of the exact increase in matrix size (storage requirements) and computation time are not included in this paper. In the past the computational requirements have been a major consideration for analysts of survey and geodetic networks. The capacities of computers have increased and will probably continue to increase. Some networks will still be too large to solve with all raw observations or single arc means. However there are now many surveys that can easily be analysed with more information in the data than just the grand means of directions.

Moreover, orientation and other nuissance parameters can be removed from the least squares network calculations using the Schreiber technique (for orientations unknowns) or nuisance parameter elimination (for orientation and other parameters) (see for example Bomford, 1985 and Caspary, 1987). These techniques remove the problem of having larger matrices to invert when extra parameters are estimated.

3.7 Statistical Considerations

When discussing the application of statistical tests, it should be kept in mind that many statistical tests are based on the presumption of random errors, strictly speaking. These test statistics and their confidence levels do not apply if non-random data are considered. Note that s_R is severely biased by systematic errors whilst s_S is less so. Now, how do statistical tests change when using single arc means instead of grand means? A prime consideration is the degrees of freedom which involves the number of observations and the number of unknowns. If the correct standard deviation is used for the input data and the appropriate degree of freedom used then there is no particular problem due to analysis procedure. However some tests and some applications of confidence levels depend on the degrees of freedom in the adjustment. Consider the example of a network with 10 sites where 8 targets are observed at each site. Each site observes 4 arcs to all targets. If the grand means are entered there are 80 measurements and 26 unknowns (10 * (2 coordinates + 1 orientation) -4 (datum defect)). So the degree of freedom is 54. If the means of each arc are entered, there are 320 measurements and 56 unknowns (10 * (2 coordinates + 4 orientations) -4 (datum defect)). So the degree of freedom is 264. If the half arcs are entered there are 640 measurements and 96 unknowns (10 * (2 coordinates + 8 orientations) -4 (datum defect)). So the degree of freedom is 544, unless additional parameters are included.

Variations in the estimated variance factor [$= v^TPv/(n-u)$] obtained from solutions using the three different types of input are not as marked as the variations to degrees of freedom highlighted in the above example. This is because even though means reduce the degrees of freedom they also reduce the sum of the weighted residuals squared (v^TPv).

The redundancy (reliability) numbers of directions improve when means of each arc or single face observations are used instead of grand means. This is just a reflection of the obvious fact that outliers are easier to isolate if you have the raw data instead of a mean or grand mean. Obviously, a mean reduces the size of any error. Note that the stochastic model of a least squares adjustment is not correct in the presence of systematic errors, unless the systematic errors are solved for or eliminated by taking means. Redundancy numbers from grand mean data are therefore significantly "purer" than those from single arc or, even worse, single face data.

4. Recommendations

Based on the errors of direction measurements presented in Table 3 it can be stated that in most least squares adjustments of networks:

* Σv^2 (and v^TPv) will be smallest if grand means are used. This is because most theodolite errors are minimised by this process. The stochastic model of random data is almost fulfilled.
* Σv^2 (and v^TPv) will be larger if single arc means are used instead of grand means. This is because circle graduation and micrometer errors will be minimised if different parts of the theodolite circle are used for each arc and the mean taken.
* Σv^2 (and v^TPv) will be largest if each half arc of data is used. This is because all those errors that are cancelled by taking two faces of observations as well as the circle graduation errors will remain in the data. The presumption of random data in the stochastic model is not fulfilled.

If grand means or the means of each arc are input then s_R should definitely not be used because it includes those errors whose effect has been cancelled by taking the mean of two faces. If circle graduation errors and other errors that vary from one arc to the next are random processes then using $s_T = s_S / \sqrt{a}$ is valid.

Considering that the use of half arcs doubles the number of observations and almost doubles the number of unknown parameters in comparison to full arcs and that the stochastic model of random errors no longer applies, due to a large number of systematic theodolite errors, the introduction of half arcs into least-squares adjustments as recommended by Patterson, is strongly discouraged.

Instead we suggest that

(i) surveyors should continue using grand means when adjusting large networks on small computers and that

(ii) surveyors should seriously consider introducing the means of single arcs rather than grand means into least squares adjustments. In doing so, separate orientation parameters should be included for each arc and $s_S S$ used as a priori standard deviations.

The use of means of single arcs (rather than grand means) greatly increases the degree of freedom of the adjustment without compromising the stochastic model seriously through the introduction of systematic errors. Data snooping is greatly facilitated, as single arc directions (rather than grand mean directions) can be eliminated if required.

If grand means are used then surveyors should continue to check the consistency of single arcs when computing the station adjustment and the associated precisions.

5. References

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